FOUR POINT CONDITION MATRICES OF EDGE WEIGHTED TREES

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Formulas for the determinant of distance matrix D_T of tree T are known in the unweighted case and in the case when the edges of T have commuting variable weights. Associated to the *Four point condition* (4PC henceforth) and a tree T are two matrices, the Max4PC$_T$ and the Min4PC$_T$. These are not full rank matrices and their rank, a basis B and formulas for the determinant when restricted to the rows and columns of B are known. In this work, we generalize both these matrices to the case when the edges of T have commuting variable weights and determine edge-weighted counterparts of known results. Results for rank are reasonably straightforward while results about the determinant of the matrix restricted to a basis are a little more involved.

This is joint work with Ali Azimi, Rakesh Jana and Mukesh Nagar.

GRAPHS FROM RINGS

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Construction of graphs from various algebraic structures has been an active field of study for the last few decades. Association of graphs with rings majorly explored in two directions. One is the study of intersection graphs of ideals of rings which we introduced in 2009. The other way is to exploit relations between elements that went around zero-divisor graphs introduced by Beck in 1988. In this talk we discuss some variations of zero-divisor graphs and a relatively new class of graphs, namely, conjugate element graphs using prime elements of the ring of integers modulo n for some natural number n.

RAMANUJAN GRAPHS (THE OPTIMAL EXPANDERS)

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Expanders are the family of graphs where each graph G on n vertices is having two competing properties (i) G is very sparse, that is, the number of edges (connections) is very less (say $<< O(n^2)$), and (ii) G is well-connected, that is, for any nonempty subset S of the vertices, the number of edges between S and its complement S' is at least $c\min(|S|,|S'|)$ where $c > 0$. The constant c is known as the isoperimetric number or Cheeger constant or expansion ratio and finding it is NP-hard. Higher c is desirable for a better expander. In the class of d-regular graphs the best possible expanders are Ramanujan graphs, these are the graphs with $\lambda_2 \leq 2\sqrt{d-1}$, where λ_2 is second largest eigenvalue of the adjacency matrix. The existence (and construction) of expanders is by no means obvious. Due to apparently conflicting features ((i), (ii)), on the one hand, the existence of such graphs is counterintuitive and, on the other hand, makes them extremely useful. The talk will be on introduction to expanders and Ramanujan graphs, and some explicit constructions of these graphs.

BLOWUP-POLYNOMIALS OF GRAPHS

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Given a finite simple connected graph $G = (V, E)$ (or even a finite metric space), we introduce a novel invariant which we call its blowup-polynomial $p_G(n_v : v \in V)$. To do so, we compute the determinant of the distance matrix of the graph blowup, obtained by taking n_v copies of the vertex v, and remove an exponential factor. In this talk, first: we show that as a function of the sizes n_v, p_G is a polynomial, is multi-affine, and is real-stable. Second: we show that the multivariate polynomial p_G fully recovers G. Third: we obtain a novel characterization of the complete multi-partite graphs, as precisely those whose "homogenized" blowup-polynomials are Lorentzian/strongly Rayleigh.

IHARA ZETA FUNCTION AND RAMANUJAN GRAPHS

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Let A be the adjacency matrix of a graph G and let the eigenvalues of A be $\lambda_0 \geq \lambda_1 \geq \lambda_2 \ldots \geq \lambda_{n-1}$. Further, let $\lambda(G)$ be the maximum of the absolute values of all the nontrivial eigenvalues, that is, $\lambda(G) = \max\limits_{i \neq 0} |\lambda_i|$. Ramanujan graph is a regular graph whose spectral gap is almost as large as possible, that is, a k regular graph satisfying $\lambda \leq 2\sqrt{k-1}$. The Ihara zeta function of a finite graph is an analogue of the Selberg zeta function, which was first introduced by Yasutaka Ihara in the context of discrete subgroups of the two-by-two p-adic special linear group. In this talk, we discuss the Ramanujan graphs and show that a regular finite graph is a Ramanujan graph if and only if its Ihara zeta function satisfies the analogue of the Riemann hypothesis as was pointed out by T. Sunada.

HYPERGRAPH SYMMETRIES AND ITS SPECTRUM

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We investigate how the structural symmetries in hypergraphs are captured by some connectivity matrices and leave traces in their spectra. The information about symmetry in a hypergraph may or may not be encoded in a matrix associated with that hypergraph. We find a family of connectivity matrices which encodes the information about some symmetries. We define a Unit as a symmetric structure where the family of matrices, including the adjacency, Laplacian, and signless Laplacian matrices of a hypergraph, captures that symmetry. We study how the spectra of these matrices reflect the existence of units in a hypergraph. Besides the spectra, units are also interrelated with hypergraph automorphisms, random walks on hypergraphs, and the chromatic number of hypergraphs.

ON THE SPECTRAL RADIUS OF BLOCK GRAPHS WITH PRESCRIBED DISSOCIATION NUMBER φ

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A connected graph is called a block graph if each of its blocks is a complete graph. Let $\mathcal{B}(\mathbf{n}, \varphi)$ be the class of block graphs on $\mathbf{n}$ vertices and prescribed dissociation number φ. We prove the existence of a block graph $\mathbf{B}_{\mathbf{n},\varphi}$ that uniquely maximizes the spectral radius $\rho(G)$ (with respect to the adjacency matrix) among all graphs G in $\mathcal{B}(\mathbf{n}, \varphi)$.

ON THE SIGNLESS LAPLACIAN SPECTRUM OF K-UNIFORM HYPERGRAPHS

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Let $\mathcal{H}$ be a connected k-uniform hypergraph on n vertices and m hyperedges. In [A. Banerjee, On the spectrum of hypergraph, Linear Algebra and its Application, 614(2021), 82–110], Anirban Banerjee introduced a new adjacency matrix for hypergraphs. In this article we consider the corresponding signless Laplacian matrix $Q(\mathcal{H})$. Let $q_{\max} = q_1 \geq \cdots \geq q_n = q_{\min}$ be the eigenvalues of $Q(\mathcal{H})$ and let $d_{\max} = d_1 \geq \cdots \geq d_n = d_{\min}$ be the degrees of $\mathcal{H}$. We obtain some upper and lower bounds for $q_{\max}$ for connected k-uniform hypergraphs. In [D. Cvetkovic, P. Rowlinson and S. K. Simic, Signless Laplacians of finite graphs, Linear Algebra and its Applications 423 (2007) 155–171], Cvetkovic et al. proved that the least eigenvalue of the signless Laplacian of a connected graph is equal to 0 if and only if the graph is bipartite. We prove that connected bipartite graphs are the only k-uniform connected hypergraphs for which the least eigenvalue of the signless Laplacian is 0. Carla Silva Oliveira et al.[Carla Silva Oliveira, Leonardo Silva de Lima, Nair Maria Maia de Abreu and Steve Kirkland,Linear Algebra and its Applications 432(2010), 2342–2351] proved that, for any graph on $n \geq 2$ vertices with at least one edge, $s_Q(G) \geq 2$ and characterized the equality case. In this article we prove that, for any connected k-uniform hypergraph $\mathcal{H}$ on $n \geq 2$ vertices with at least one hyperedge, $s_Q(\mathcal{H}) > 1$.

A CHARACTERIZATION OF STATE TRANSFER ON DOUBLE SUBDIVIDED STARS

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A subdivided star $SK_{1,l}$ is obtained by identifying exactly one pendant vertex from l copies of the path P_3. This study is on the existence of quantum state transfer on double subdivided star $T_{l,m}$ which is a pair of subdivided stars $SK_{1,l}$ and $SK_{1,m}$ joined by an edge to the respective coalescence vertices. Using the Galois group of the characteristic polynomial of $T_{l,m}$, we analyze the linear independence of its eigenvalues which uncovers no perfect state transfer in double subdivided stars when considering the adjacency matrix as the Hamiltonian of corresponding quantum system. Finally, we provide a complete characterization on double subdivided stars exhibiting pretty good state transfer.

Keywords: Spectra of graphs, Galois group, Perfect state transfer, Pretty good state transfer.

THE HERZOG-HIBI-OHSUGI CONJECTURE ON POWERS OF VERTEX COVER IDEALS OF CHORDAL GRAPHS

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In 1999, Herzog and Hibi introduced the concept of componentwise linear ideals. This concept found great significance in algebraic combinatorics and the field of commutative algebra. In 2011, Herzog, Hibi, and Ohsugi proposed a conjecture stating that the cover ideal of a chordal graph satisfies the property of being componentwise linear for all powers greater than or equal to 1. This talk will present a proof of this conjecture, specifically for the case of forest graphs. Using techniques from commutative algebra and graph theory, we will demonstrate how the cover ideal of a forest graph can be shown to be componentwise linear for all powers. This result has potential implications in understanding the structure and properties of cover ideals in other types of graphs as well.

ON THE MINIMUM CUT-SETS OF THE POWER GRAPH OF A FINITE CYCLIC GROUP

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The power graph $\mathcal{P}(G)$ of a finite group G is the simple graph with vertex set G, in which two distinct vertices are adjacent if one of them is a power of the other. For an integer $n \geq 2$, let C_n denote the cyclic group of order n and let r be the number of distinct prime divisors of n. In this talk, for $r \geq 4$, we identify certain cut-sets of $\mathcal{P}(C_n)$ such that any minimum cut-set of $\mathcal{P}(C_n)$ must be one of them.

THE BIPARTITE LAPLACIAN MATRIX OF A NONSINGULAR TREE

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Similar to the bipartite adjacency matrix (J. A. Bondy and U. S. R. Murty. *Graph theory*, Springer-Verlag London, New York, 2008), we define the *bipartite distance matrix* of a bipartite graph with a unique perfect matching. The *bipartite distance matrix* $B(G)$ of a bipartite graph G with a unique perfect matching on $2p$ vertices is a $p \times p$ matrix whose (i, j)th entry is the distance between vertices l_i and r_j, where $L := \{l_1, \ldots, l_p\}$, $R := \{r_1, \ldots, r_p\}$ is a vertex bipartition of G. We observe that $\det B(G)$ is always a multiple of 2^{p-1}. Based on this observation, we define the *bipartite distance index* of G as $\mathtt{bd}(G) := \det B(G)/(-2)^{p-1}$.

We demonstrate that the bipartite distance index of a nonsingular tree T satisfies an interesting inclusion-exclusion type of principle at any matching edge of the tree. Furthermore, we fully characterize the bipartite distance index of a nonsingular tree T by the structure of T through what we term f-alternating sums.

The study of the inverse of the bipartite distance matrix leads to an unexpected generalization of the usual Laplacian matrix for a tree, referred to as the *bipartite Laplacian matrix*. This generalized Laplacian matrix is usually not symmetric but it shares many properties with the usual Laplacian matrix. We study some of the fundamental properties of the bipartite Laplacian matrix and compare them with those of the usual Laplacian matrix. Lastly, we present a combinatorial description of all minors of the bipartite Laplacian matrix for a nonsingular tree.

MINIMIZING ALGEBRAIC CONNETIVITY OVER GRAPHS CONSTRUCTED WITH GIVEN BLOCKS

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We call a block pendant if it has exactly one cut vertex. Suppose that we are given a collection blocks and we construct all possible connected graphs with these blocks keeping the number of pendant blocks fixed. In this talk, we describe the structure of the graphs that minimize the algebraic connectivity among all such graphs. As an application, we conclude that over all such graphs made with the given blocks, the algebraic connectivity is minimum for a graph whose block structure is a path.

ECCENTRICITY MATRIX AND CORONA PRODUCT OF GRAPHS - AN OVERVIEW

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In a connected graph, eccentricity of a vertex $u \in G$, $(e(u))$, is defined as the maximum distance from u to any other vertex $v \in G$ in the graph. The eccentricity matrix, $\varepsilon(G)$ is derived from the distance matrix whose $(u, v)^{th}$ entry is equal to $d(u, v)$ if $d(u, v)$ is the minimum of $\{e(u), e(v)\}$, and is zero otherwise. Note that $d(u, v)$ is the distance between the vertices u and v. A graph G is said to be self centered if the eccentricity of each vertex of G is same. Assume that G and H are two finite graphs on m and n vertices, respectively. Let $G \circ H$ be the corona of G and H, constructed by keeping a single copy of G and m copies of H, and then connecting each vertex of the j^{th} copy of H to the j^{th} vertex of G. In this talk, we focus on recent advances in the study of eccentricity matrix of a graph. Further, we discuss and analyze the spectra of eccentricity matrices obtained by corona product of graphs G and H, where G is a self-centered and connected graph.

This is a joint work with Ms. Smrati Pandey, Department of Mathematical Sciences, IIT(BHU) Varanasi.

ON THE ECCENTRICITY MATRICES OF TREES: INERTIA AND SPECTRAL SYMMETRY

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The *eccentricity matrix* $\mathcal{E}(G)$ of a connected graph G is obtained from the distance matrix of G by keeping the largest nonzero entries in each row and each column, and leaving zeros in the remaining ones. The eigenvalues of $\mathcal{E}(G)$ are the $\mathcal{E}$*-eigenvalues* of G. It is well known that the distance matrices of tress are invertible and the determinant depends only on the number of vertices. We show that the eccentricity matrix of tree T is invertible if and only if either T is star or P_4. Also we show that any tree with odd diameter has 4 distinct $\mathcal{E}$-eigenvalues, and any tree with even diameter has the same number of positive and negative $\mathcal{E}$-eigenvalues (which is equal to the number of 'diametrically distinguished' vertices). Finally, we will discuss about the trees with $\mathcal{E}$-eigenvalues are symmetric with respect to the origin.

SINGULAR GRAPHS AND THE RECIPROCAL EIGENVALUE PROPERTY

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Let G be a simple connected graph and $A(G)$ be its adjacency matrix. A nonsingular graph G is said to have the reciprocal eigenvalue property if the reciprocal of each eigenvalue of G is also an eigenvalue. Furthermore, if each eigenvalue of G and its reciprocal have the same multiplicity, then G is said to have the strong reciprocal eigenvalue property. Such graphs exist and have been studied in the past. A general question remained open. Can there be a graph that has at least one zero eigenvalue and whose nonzero eigenvalues satisfy the reciprocal eigenvalue property? We first prove that there is no such nontrivial tree. Suppose that G is singular and the characteristic polynomial of G is $x^{n-k}(x^k + a_1x^{k-1} + \cdots + a_k)$. Assume that $A(G)$ has rank k, so that $a_k \neq 0$. Can we ever have $|a_k| = 1$? The answer turns out to be negative. As an application, we prove that there is no nontrivial singular graph whose nonzero eigenvalues satisfy the reciprocal eigenvalue property.

This talk is based on joint work with Debabrota Mondal, Sukanta Pati, and Kuldeep Sarma.